Transverse beam temperature effects on mixed Two-Stream/Filamentation unstable modes

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Abstract

We investigate transverse beam temperature effects on the coupling between Two-Stream and Filamentation instabilities for an electron beam passing through a hot plasma with return current. By denoting $\theta_k$, the angle between the wave vector and the beam, transverse beam temperature effects are important for $\theta_k$ larger than a critical angle $\theta_c$ which can be determined exactly, the resulting effects being a damping of the instabilities. On the other hand, effects are small for $\theta_k < \theta_c$. In particular, the absolute maximum growth rate is found for a wave vector in this region and is therefore not damped by transverse beam temperature.

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1. Introduction

The Fast Ignition Scenario (FIS) involves the interaction of an electron beam (velocity $V_b$) and return current with a hot and dense plasma. The resulting system is known to undergo various linear electromagnetic instabilities: the Two-Stream instability has $k \parallel V_b$ and $k \parallel E$ (the wave electric field). We shall identify as Weibel instability unstable modes with $k \parallel V_b$ (as in the original Weibel work [1]) but $k \perp E$, and label “Filamentation” instability unstable modes with $k \perp V_b$ and $k \perp E$. A large amount of work has recently been devoted to the subject and some authors [2] have pointed out the need to analyze the coupling between Two-Stream and Filamentation instabilities. Indeed, Two-Stream and Filamentation instabilities correspond to extreme wave vector orientations and it is relevant to investigate instabilities all over the $k$ space. In a recent paper [3], we did so for a cold beam interacting with a
hot plasma. However, it is known that the transverse beam temperature reduces Filamentation instability [2,4] whereas it leaves the Two-Stream instability almost unaffected [4]. It is therefore important to determine how the oblique unstable modes are modified. Only the non-relativistic beam case will be treated here, as a starting point to the full relativistic treatment necessary for FIS.

2. Theoretical framework

We consider a homogeneous, spatially infinite, collisionless and unmagnetized plasma whose dynamics is ruled by the Vlasov–Maxwell equations for the electronic distribution function \( f(\mathbf{v}, \mathbf{r}, t) \) and the electromagnetic field. Ions are supposed to form a fixed neutralizing background. Within the linear approximation, the dielectric tensor elements are [5,6]

\[
\epsilon_{\alpha\beta} = \delta_{\alpha\beta} \left( 1 - \frac{\omega_p^2}{\omega^2} \right) + \frac{\omega_p^2}{n_e \omega^2} \int v_{\alpha} v_{\beta} \mathbf{k} \cdot \nabla f_0 \frac{\partial}{\partial v_{\gamma}} \mathbf{v} \, d^3 \mathbf{v}.
\]  

(1)

The plasma frequency is given by \( \omega_p = \sqrt{4 \pi n_e e^2/m_e} \), with \( n_e \) electron density and \( m_e \) electron mass. The electron beam velocity is aligned with the \( z \)-axis and the wave vector lies in the \( (x, z) \) plane. We define \( \theta_b = (\mathbf{k}, \mathbf{e}_z) \) so that the Two-Stream configuration corresponds to \( \theta_b = 0 \) and Filamentation to \( \theta_b = \pi/2 \). The equilibrium distribution function is \( f_0 = f_0^p + f_0^b \) with \( (i = p, b) \),

\[
f_0^i = \frac{n_i}{2V_i} \left[ \Theta(v_x + V_i) - \Theta(v_x - V_i) \right] \times \delta(v_y) \delta(v_z + V_i)
\]  

(2)

where \( n_{p,b} \) and \( V_{b,p} \) are the beam and plasma electronic density and electronic current velocity respectively. We take \( n_pV_p = -n_bV_b \), which implies current neutralization. \( \Theta(x) \) denotes the Heaviside step function. Such water-bag distributions provide a classical tool to derive analytical results for temperature effects [2,7] and we introduce it here for a similar purpose. One can note that the system is cold in the \( y \) direction. This has no bearing on the present study because we restrict to the Two-Stream/Filamentation (TSF) branch, but such an approximation fails to recover Weibel-like modes (see Ref. [3]). Finally, let us introduce the dimensionless variables \( \Omega = \omega/\omega_p \), \( Z = kV_b/\omega_p \), \( \alpha = n_b/n_p \), \( \beta = V_b/c \), \( \rho_p = V_{bp}/V_b \), \( \rho_p = V_{bp}/V_b \). For a distribution function which can be put under the form \( \sum_j f_j(v_x)g_j(v_y)h_j(v_z) \) with \( g_j \) and \( h_j \) even functions of \( v_x \) and \( v_y \), the dispersion equation for the TFS branch is [3] (\( \eta \equiv \omega/c \))

\[
(\eta^2 \epsilon_{xx} - k_z^2)(\eta^2 \epsilon_{zz} - k_z^2) - (\eta^2 \epsilon_{xx} - k_z \epsilon_{xz})^2 = 0.
\]  

(3)

3. Analysis

For the Two-Stream instability, we set \( k_z = 0 \) in Eqs. (1), (3). In the limit \( \alpha \ll 1 \), modes are found unstable for \( Z < 1 + 3/2\alpha_{1/3} \). The maximum growth rate \( \delta_{m0} = \sqrt{3/24/3} \alpha_{1/3} \) is reached for \( Z \sim 1 \). This growth rate bears no transverse temperature effects, whether it be plasma or beam temperature. For the Filamentation instability, we set \( k_z = 0 \) in Eqs. (1), (3) and calculate the dispersion equation. The electric field of the mode is found aligned with the \( z \)-axis [3] so that the waves are transverse, as is well known. In the limit \( \alpha, \rho_p \ll 1 \) and \( \rho_b/\rho_p \ll 1 \), modes are unstable for

\[
Z \lesssim \frac{\beta}{\rho_p} \left( 1 - \frac{\rho_b^2}{2\alpha \rho_p^2} \right).
\]  

(4)

With \( \alpha \ll 1 \), Eq. (4) shows that the reduction of the instability domain is very fast as soon as \( \rho_b > 0 \). The maximum growth rate \( \delta_{m} \) vanishes for \( \rho_b \sim \sqrt{\alpha} \) and is very well fitted by

\[
\delta_{m} \sim \beta \sqrt{\alpha} \left( 1 - \frac{\rho_b}{\sqrt{\alpha}} \right).
\]  

(5)

We found no transverse beam temperature correction to the Two-Stream configuration while effects are important for Filamentation. Our goal from now on is to find out when the transverse beam temperature becomes an important factor as the wave vector departs from the beam axis. An analysis involving the singularities of the dispersion function reveals a critical angle when two
This quantity is exact and can be recovered using the longitudinal (or electrostatic, $k \parallel E$) approximation to calculate the growth rate even though such an approximation obviously fails to recover transverse Filamentation modes. It turns out that this angle divides the $k$ space between a Filamentation-like and a Two-Stream-like region. Also, modes are unstable for all $k$ in this direction. The transverse temperature effect is well illustrated by Figs. 1(a–c). For the parameters we chose, the Filamentation growth rate vanishes for a transverse beam temperature $\rho_b \sim 0.3$ and even with the smaller temperature chosen for the plot, $\rho_b = \frac{1}{30}$, Filamentation damping is obvious in Figs. 1(a and c) in the Filamentation-like region. On the other hand, the Two-Stream-like region located below the $\theta_c$ line is weakly affected and there is absolutely no transverse beam temperature effect for $\theta_k = 0$. The longitudinal calculation (Fig. 1(b)) and the full electromagnetic calculation (Fig. 1(c)) are almost identical because transverse temperature damps the growth rate precisely where longitudinal calculation should fail, namely in the Filamentation region beyond $\theta_c$. The asymptotic growth rate in the $\theta_c$ direction can be evaluated in the $\rho_p, \rho_b \ll 1$ limit and one finds $\delta \theta_k \sim \beta \sqrt{\alpha}$.

4. Conclusion

We investigated the transverse beam temperature effect for a wave vector arbitrarily oriented. We found that the transverse temperature stabilizes the system for $\theta_k > \theta_c$. It does not stabilize modes below $\theta_c$ and leaves absolutely unchanged the most unstable mode, still reached for $\theta_k = 0$. We are currently investigating the relativistic beam regime to get closer to FIS conditions.

References