Elastoplastic effects on the Rayleigh-Taylor instability in an accelerated solid slab

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Abstract. The instability of an accelerated solid slab with elastoplastic properties is analysed by means of numerical simulations. The study is performed in the framework of experiments to be carried out at the future synchrotron facility SIS100 at the Gesellschaft für Schwerionenforschung (GSI), Darmstadt, which consider the implosion of a heavy pusher layer driven by the expansion of a material heated by an intense beam of heavy ions. We have found that for the megabar pressures involved in these experiments, the solid pusher may retain elastoplastic properties that are beneficial for the stabilization of the shortest perturbation wavelengths.

PACS. 52.35.Py Macroinstabilities (hydromagnetic, e.g., kink, fire-hose, mirror, ballooning, tearing, trapped-particle, flute, Rayleigh-Taylor, etc.) – 52.50.Gj Plasma heating by particle beams

1 Introduction

Several plasma physics experiments that will be carried out at the future synchrotron facility SIS100 at the Gesellschaft für Schwerionenforschung (GSI), Darmstadt, involve the study of matter under extreme conditions of pressure and density. A typical experiment considers the implosion of a multilayer cylindrical target that contains a material sample (for instance, hydrogen) in the axial region [1–3]. The target is axially irradiated from one side of the cylinder by an intense heavy ion beam with an annular focal spot (Fig. 1). The inner radius of the ring shaped focal spot of this hollow beam is larger than the radius of the sample in order to avoid the direct heating of the sample material. Besides, the target length is assumed to be shorter than the ion range so that the absorption region is heated with an adequate axial symmetry. When the annular region of the absorber is heated by the ion beam, it expands thereby pushing the inner layers of the target (the pusher) and compressing the sample material in the axial region.

Implosion of such a target configuration has been recently studied by means of analytical models and numerical simulations [1–3]. In particular, we have shown that such a configuration is very suitable for an experiment dedicated to the study of the hydrogen metallization problem [4,5]. However, a challenging problem in this target design is the generation of an annular focal spot of the ion beam. A proposal to produce such an annular spot considers a high-frequency rf-wobbler that will rotate the ion beam with a rotation frequency of the order of GHz [6]. We have recently analyzed the symmetry constrains imposed by such a wobbler system and we have found that a level of symmetry of 1% in the driving pressure on the pusher surface can be achieved provided that the beam performs about ten revolutions during the pulse duration [7,8]. This value is equal to the expected intrinsic symmetry level of the system and hence a larger number of revolutions cannot improve the implosion symmetry any further.

Although an acceptable symmetry level can be reached with the rotating beam, the stability of the pusher that drives the implosion still remains another issue of possible concern. In fact since the relatively low temperatures in the absorber region prevent the ablation of the pusher material, there are no apparent mechanisms that could reduce the growth rate of the Rayleigh-Taylor (RT) instability seeded by the asymmetries in the pusher-absorber interface [9,10]. As is well known the RT instabilities arise when a higher density fluid lies above a lower density fluid in a gravitational field or, equivalently, when the low density fluid of the target absorber pushes the higher density fluid.

Fig. 1. Beam-target geometry.
pusher. Nevertheless, we have noticed from the numerical simulations that, for driving pressures of the order of few megabars, the pusher material remains in a solid or liquid state which may still retain some of the elastoplastic properties of the material. Previous works have shown that under these conditions such elastoplastic properties can provide a stabilizing mechanism that is certainly beneficial for the previously mentioned experiments [11–16].

In the present work we have performed a series of two-dimensional (2D) simulations in order to study the influence of the material elastoplastic behavior on the growth rate of the RT instability of accelerated planar solids. This behavior is defined by the yield stress \( Y \) and the shear modulus \( G \) of the material. In the present case, however, they must be taken as parameters because their real values for solids under megabar pressures are unknown. Experiments like those presented in reference [17] could be used in the future as a tool for determining such values.

In Section 2.1 we summarize the main features of the physical model included in the code ABAQUS [18] that we have used for the description of the elastoplastic behaviour of the pusher material. In Section 2.2 we have simulated a few cases for which experimental results are available in order to validate the physical model of Section 2.1. In Section 3 we perform our numerical study of the instability of an accelerated gold slab under conditions in which the elastoplastic behaviour is relevant. Conclusions drawn from this work are noted in Section 4.

2 The physical model

2.1 Main equations and assumptions

We have performed this numerical analysis using an explicit version of the ABAQUS [18] finite element code which is suitable for modeling fast transient phenomena. This code is based on a central difference scheme for the time integration of the equations of motion. The algorithm has second-order accuracy and is conditionally stable [19], so that the time step is fixed according to the Courant-Friedrich-Levy (CFL) criteria.

The material description is characterized by the volumetric and deviatoric parts and it is assumed that these two behaviours are not coupled. The material hydrostatic behaviour is defined by the Mie-Grüneisen equation of state [20]:

\[
P - P_h = \Gamma_0 g (E - E_h),
\]

where \( P \) and \( E \) are the pressure and the specific internal energy and \( P_h \) and \( E_h \) are the Hugoniot pressure and Hugoniot specific internal energy, respectively. The latter are functions of the density \( \varrho \) only. Also \( \Gamma \) is the Grüneisen ratio and is defined in the usual manner:

\[
\Gamma = \Gamma_0 \frac{\varrho_0}{\varrho},
\]

where \( \Gamma_0 \) is a constant characteristic of the material and \( \varrho_0 \) is the reference density.

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<th>Table 1. Parameters of the model.</th>
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<td>( \varrho ) (g/cm(^3))</td>
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<td>( G ) (GPa)</td>
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<td>( Y ) (GPa)</td>
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On the other hand, the equation of energy conservation is:

\[
\frac{\partial E}{\partial t} = P \frac{\partial \varrho}{\varrho} + S : \dot{\varepsilon} + \varrho \dot{Q},
\]

where \( S \) is the deviatoric stress tensor, \( \dot{\varepsilon} \) is the deviatoric strain rate tensor and \( \dot{Q} \) is the heat rate per unit mass.

The equation of state and the equation of energy conservation are coupled through the pressure and the internal energy. The Hugoniots relate the pressure, internal energy and density behind the shock waves to the corresponding quantities in front of them in terms of the shock velocity \( u_s \) and the particle velocity \( u_p \). As is well known, the Hugoniots of many materials can be adequately represented by the following linear relationship [20,21]:

\[
u_s = c_0 + s u_p,
\]

where \( c_0 \) and \( s \) are fitting parameters that depend on the considered material.

The deviatoric part of the stress tensor describing the elastoplastic behaviour of the material is modeled by using a perfectly plastic model in which the yield surface that defines the stress state of material points behaving plastically does not change with the plastic deformation. This yield surface follows from a von Mises criterion and an associative rule is assumed to define the plastic flow [22]. This simple elastoplastic model only needs two parameters: the shear modulus \( G \) that defines the slope of the linear relationship between deviatoric stresses and deviatoric elastic strains, and the yield stress \( Y \) that defines the onset of the plastic range. Although much more complex models including strain hardening, thermal softening or strain rate dependence could be used, here we have used this simple two-parameters model because it seems to be enough for our present purposes of studying the stabilization effects of the elastoplastic behaviour of the RT instability on an accelerated solid slab (see Sect. 2.2).

In the present work we have analysed the growth rate of the RT instability in both aluminium and gold planar slabs. The parameters required by the model are summarized in Table 1 for these two materials. The values of the constants \( c_0 \), \( s \) and \( \Gamma_0 \) that define the volumetric response of the material have been obtained from reference [23]. For all the cases studied in this work the values of the shear modulus \( G \) and the yield stress \( Y \) have been taken to be of the order of their conventional values.

The simulated planar slab has a thickness \( h \) and it has been spatially discretized with quadrilateral elements and
a sketch of it is shown in Figure 2. The two-dimensional simulations performed with the ABAQUS code account for an external uniform pressure applied at the upper solid-vacuum surface which is located at \( y = h \) at time \( t = 0 \). In order to initialise the hydrodynamic instability, the position of the nodes of the entire mesh has been perturbed by the quantity \( \delta(x, y) = a_0 \frac{t}{\delta} \cos(kx) \) (\( x \) and \( y \) are the horizontal and vertical coordinates respectively, \( k = 2\pi/\lambda \) is the wavenumber, \( \lambda \) is the wavelength, and \( a_0 \) is the maximum amplitude). It follows that the amplitude has its maximum value located at \( y = h \) and then it decreases linearly with the coordinate \( y \) up to \( y = 0 \) where it vanishes. In the lateral direction (\( x \)-axis) we consider a region of one perturbation wavelength \( \lambda \) in length (Fig. 2) and then we apply periodical boundary conditions in order to simulate a media with infinite lateral dimensions.

Due to the applied pressure, the solid slab is accelerated and the upper solid-vacuum interface becomes RT unstable with the resulting growth of the amplitude of the perturbation with time.

2.2 Comparison with experimental results

In order validate the numerical model we have considered the experimental results by Barnes et al. [11,13] in a similar manner as in the work by Swegle and Robinson [14]. The experiments were conducted using aluminium plates of thickness \( h = 2.54 \) mm and they were initialised with sinusoidal perturbations of different wavelengths \( \lambda \) and amplitudes \( a_0 \). We have used the data obtained in three different experimental conditions. Two cases were perturbed with a wavelength \( \lambda = 2h \) and amplitudes \( a_0 = 100 \) \( \mu \)m and \( a_0 = 50 \) \( \mu \)m, and the third case with \( \lambda = h \) and \( a_0 = 50 \) \( \mu \)m. In the experiments, the planar targets were smoothly accelerated by using a gas detonation.

We carried out a set of simulations to compare the collected experimental data with our numerical results. A pressure applied at the perturbed surface has been used to generate the shock launched into the solid planar plate. The simulated slab was discretized with \( 80 \times 40 \) elements. Our external applied pressure grows linearly in time reaching the maximum value of 10 GPa at \( t = 1.4 \) \( \mu \)s and then it decreases almost linearly up to a value of 2 GPa at \( t = 6 \) \( \mu \)s. Such a pressure profile is similar to that one used by Swegle and Robinson [14]. The simulations give the evolution of the perturbation amplitude as a function of time as shown in Figure 3 where the experimental data have also been indicated with different symbols. These simulations were performed using several values of the yield stress \( Y \). The experimental data were well reproduced by considering yield stresses \( Y \) ranging between 0.30 GPa and 0.35 GPa, in agreement with the results of reference [14].

3 Study of the RT instability with elastoplastic material effects

Here we present a parametric analysis for the study of the influence of the elastoplastic behaviour on the growth rate of the instability in solid slabs. Several calculations varying the yield strength \( Y \) and the shear modulus \( G \) have been performed by using gold slabs initially perturbed with different wavelengths \( \lambda \), amplitudes \( a_0 \) and with an assumed slab thickness of \( h = 200 \) \( \mu \)m. The results are compared with those corresponding to the classical case, that is, when the slab behaves like an inviscid and incompressible fluid. The latter situation represents the worst case with the fastest growth of the perturbations. In fact, as it is well known from the linear theory of the RT instability of stratified fluids, the perturbation amplitude \( a \) grows exponentially in time and it is characterized by the growth rate \( \gamma \) which is a function of the perturbation wavelength \( \lambda \) [9,10]:

\[
a = a_0 e^{\gamma t}.
\]
If the stress gradient is due to a gravitational acceleration \( g \), the exponential growth rate is given by:

\[
\gamma = \sqrt{A_T k g_0},
\]

where \( A_T \) is the Atwood number and in the case of a fluid-vacuum interface \( A_T = 1 \). For the case in which the stress gradient is produced by means of a driving pressure \( P \), the exponential growth rate results to be:

\[
\gamma = \sqrt{\frac{k P}{\rho_0 h}}.
\]

To simulate the conditions in which equation (5) is applicable, it is necessary to take some care in the calculations because the exponential growth rate given by this equation assumes an initially steady-stress distribution. In our calculations, however, we consider an initially stress-free slab. If the applied driving pressure rises slowly enough in a time that is long in comparison with the transit time of a sound wave across the slab, the steady-state stress distribution will be well approximated. For this scope we have used a pressure profile with a rise time of 100 ns after which the load is kept constant and equal to 40 GPa.

It is also worth mentioning that the exponential growth given by equation (5) is valid for the linear phase of the growth provided that the growth rate is independent of the perturbation amplitude. This is just the case for the classical RT instability in a fluid but the situation may be different for an accelerated slab with elastoplastic properties [15,16]. Thus in order to check the possible dependence of the growth rate on the amplitude of the perturbation for a range of parameter values in which we are interested, we have started our analysis by considering a gold planar slab perturbed with sinusoidal perturbations of wavelength \( \lambda = h = 200 \)\( \mu m \) which has different initial amplitudes \( a_0 = 1, 10^{-1}, 10^{-2}, \) and \( 10^{-3} \)\( \mu m \) so that it is always \( a_0 \ll \lambda \) and we should be in the linear regime. For all the cases the elastoplastic parameters of the material have been taken as \( Y = 8 \times 10^7 \) Pa and \( G = 30 \) GPa, and the slab has been discretized with \( 200 \times 200 \) elements.

In Figure 4 we show the time evolution of the different amplitudes. It is seen that after \( t = 100 \) ns when the rising phase of the pressure has finished and during the time in which the perturbation remains smaller than the perturbation wavelength, the slopes of the curves are practically the same. This indicates that in the regime in which we are interested, the dependence of the growth rate on the amplitude can be neglected and the perturbation growth can be taken as exponential. So that the growth rate can be determined from the slope of the curves of amplitude versus time as in the classical fluid case.

Taking into account the previous result we can proceed to study the effect of the material elastoplastic behaviour on the instability growth rate in a framework of relevance for the target implosion experiment discussed in the introduction. To this scope we have simulated several cases for a gold slab with initial perturbations of different wavelengths, \( \lambda = h, h/2, h/4, h/8, h/12 \) and \( h/16 \). For all the cases, the number of elements of the mesh is the same in the \( x \)-axis and, in order to keep the same aspect ratio of the elements in all the calculations, their number along the \( y \)-axis is variable. Thus the meshes vary from 20 \( \times \) 20 elements for the case with \( \lambda = h \), to 20 \( \times \) 320 for the case with \( \lambda \approx h/16 \). We have performed these calculations for different values of the yield stress \( Y \) between \( 2 \times 10^7 \) Pa and \( 10^8 \) Pa. As we have already mentioned the actual value of the yield stress for a solid under a pressure of 40 GPa is not known and therefore we have taken values below that one corresponding to the yield stress under static conditions (\( Y \approx 10^8 \) Pa for gold). In particular, we recover the classical growth rate for a fluid given by equation (7) when the yield stress is taken to be \( Y = 0 \) even if the fluid behaviour is already reached when \( Y \approx 2 \times 10^7 \) Pa indicating that for smaller values, the elastoplastic effects on the RT instability seems to be negligible.

We have also performed a few calculations with different values of the shear modulus \( G \) for a given value of the yield stress \( Y \) and we have found that the value of \( G \) has no significant effect on the instability growth rate. Therefore, in the calculations presented below we have taken a constant value of \( G = 30 \) GPa.

The results of the numerical calculations are shown in Figure 5 where we show the time evolution of the amplitude for the case with \( Y = 4 \times 10^7 \) Pa and for different perturbation wavelengths \( \lambda \). Here we have taken the conditions at \( t = 0 \) in such a way that we have the same amplitudes at \( t = 100 \) ns (initial conditions), that is, \( a(t = 100 \) ns) = 0.15 \( \mu m \). This initial value of the amplitude is small enough to ensure that the instability develops in the linear regime. As one can see, the instability growth rate decreases with the perturbation wavelength so that it becomes practically equal to zero for \( \lambda = h/16 \), as it is indicated by the slope of the corresponding curve.

In order to ensure that the spatial discretization required by the numerical calculations does not introduce secondary modes that could affect the growth rate calculations shown in Figure 5, we have performed a spectral...
Fig. 5. Amplitude $a$ as a function of time for a gold planar slab with $Y = 4 \times 10^7$ Pa and $G = 30$ GPa. The simulations have been initialised with different wavelength $\lambda$. The upper curves correspond to $\lambda = h$, $h/2$ and $h/4$ and the lower curves correspond to $\lambda = h/8$, $h/12$ and $h/16$. For each case the initial amplitude $a_0$ has been adjusted in order to have the same amplitude $a(t = 100 \text{ ns}) = 0.15 \mu m$ at 100 ns corresponding to the end of the rising phase of the applied pressure.

Fig. 6. Fourier analysis of the position $y(x,t)$ of the perturbed surface performed by giving the time evolution of the harmonic amplitudes. The normalised amplitudes as a function of time are shown for the case of $\lambda = h$ and $Y = 4 \times 10^7$ Pa. In this figure we can see the time evolution of the different harmonics of the perturbed surface and, as it is seen, the secondary harmonics introduced by the discretization are much smaller than the main harmonic.

Now, we can compute the instability growth rate as a function of the perturbation wavelength and for different values of the yield strength $Y$. The results are summarized in Figure 7. As we can see, for the lowest value of $Y$ we have considered, $Y = 2 \times 10^7$ Pa, the growth rate is very similar to the classical case corresponding to a fluid, $\gamma = \sqrt{2\pi g/\lambda}$. However, for higher values of $Y$ the growth rate is reduced as a consequence of the elastoplastic behaviour of the material and it becomes practically zero for relatively short perturbation wavelengths. Thus, the elastoplastic properties of the material produce a stabilizing effect for all the perturbation wavelengths shorter than some cut-off value similar to the situations in which ablation is present. We see that even a relatively strong reduction of $Y$ by a factor of 2.5 with respect to its standard solid value ($Y \approx 10^8$ Pa) leads to a growth rate reduction of almost two orders of magnitude compared with the classical value for $\lambda \leq 10 \mu m$.

4 Conclusions

We have studied the RT instability of an accelerated solid slab in the framework of the target design considered for the experiments to be carried out in the future facility SIS100 at the GSI Darmstadt. In these experiments the annular absorber region of a multilayered cylindrical target will be heated by an intense beam of heavy ions, thus creating megabar pressures to implode a solid pusher that surrounds a material sample placed in the axial region. The configuration in which a low density absorber is pushing the denser pusher is hydrodynamically unstable, and the generation of such an annular region by means of the rotation of the ion beam can create the initial asymmetries able to seed the instabilities. The relatively low temperatures and low pressures in the absorber prevent any ablative mechanism of stabilization but these conditions may allow for conserving the elastoplastic properties of the pusher material. Although the actual values of the yield stress under such conditions are not known at present, we should expect a reduction of its value with respect to the value for a solid under standard conditions, as the pressure applied on the pusher increases so that, for very high pressures, the material behaves like a standard fluid. In the intermediate regime in which the experiments at the GSI Darmstadt will be carried out, the elastoplastic properties of the pusher material cause a significant reduction of the instability growth rate compared with the pure fluid case. According to the present results, if the
yield stress remains above $3 - 4 \times 10^7$ Pa all the perturbation wavelengths shorter than $10 \mu$m will be stabilised. For the typical targets considered for the GSI Darmstadt experiments, it means that all the modes $l = 2\pi R/\lambda$ higher than $l_{\text{max}} = 100$ are stabilized ($R$ is the pusher radius by the time of maximum acceleration). Therefore, the wobbler system currently under design for rotating the beam that will generate the annular absorber region must be fast enough not only for reducing the symmetry to an acceptable level but also for eliminating the lowest modes. This fact introduces a new constraint that must be taken into account in the design of such a wobbler system and it will be considered in a future study.

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