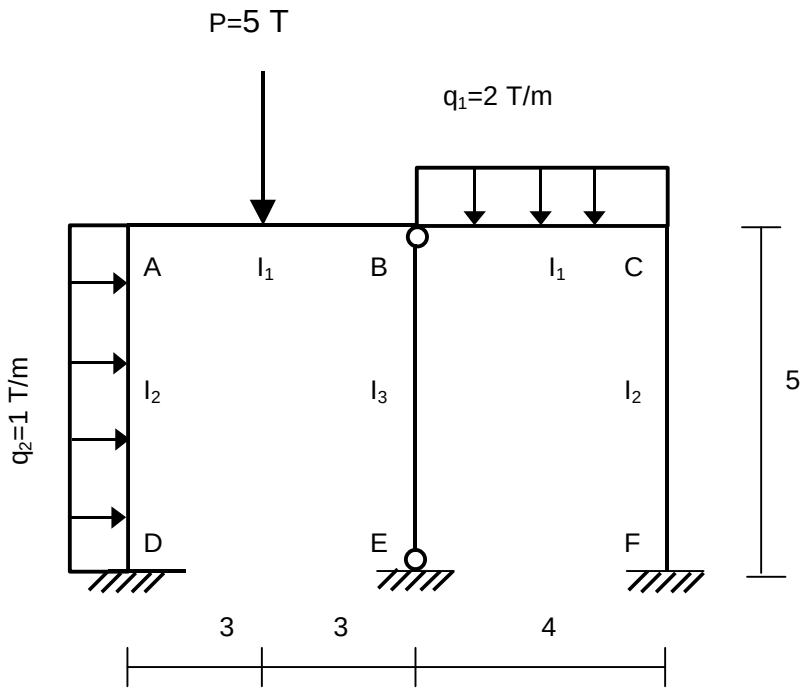
Hallar por el método de Cross los diagramas de momentos flectores y esfuerzos cortantes, así como las reacciones de todas las barras del pórtico de la figura.

La relación entre los momentos de inercia de las barras es:

$$I_1 = 2 \cdot I_2 = 3 \cdot I_3$$



1º. Determinamos los coeficientes elásticos (β_i, K_i y r_i).

Nudo A

$$K_{AD} = \frac{4 \cdot E \cdot I_2}{5} = 0.8 \cdot E \cdot I_2$$

$$K_{AB} = \frac{4 \cdot E \cdot I_1}{6} = 1.33 \cdot E \cdot I_2$$

$$\beta_{AB} = \frac{1}{2}$$

$$\beta_{AD} = \frac{1}{2}$$

$$r_{AD} = \frac{K_{AD}}{K_{AD} + K_{AB}} = \frac{0.8}{0.8 + 1.33} = 0.38$$

$$r_{AB} = \frac{K_{AB}}{K_{AD} + K_{AB}} = \frac{1.33}{0.8 + 1.33} = 0.62$$

Nudo B

$$K_{BA} = \frac{4 \cdot E \cdot I_1}{6} = 0.67 \cdot E \cdot I_1$$

$$K_{BC} = \frac{4 \cdot E \cdot I_1}{4} = E \cdot I_1$$

$$K_{BE} = 0$$

$$\beta_{BA} = \frac{1}{2}$$

$$\beta_{BC} = \frac{1}{2}$$

$$\beta_{BE} = 0$$

$$r_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC} + K_{BE}} = \frac{0.67}{0.67 + 1 + 0} = 0.40$$

$$r_{BC} = \frac{K_{BC}}{K_{BA} + K_{BC} + K_{BE}} = \frac{1}{0.67 + 1 + 0} = 0.60$$

$$r_{BE} = \frac{K_{BA}}{K_{BA} + K_{BC} + K_{BE}} = 0$$

Nudo C

$$K_{CB} = \frac{4 \cdot E \cdot I_1}{4} = 2 \cdot E \cdot I_2$$

$$K_{CF} = \frac{4 \cdot E \cdot I_2}{5} = 0.8 \cdot E \cdot I_2$$

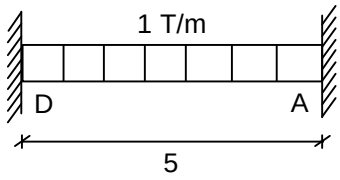
$$\beta_{CB} = \frac{1}{2}$$

$$\beta_{CF} = \frac{1}{2}$$

$$r_{CB} = \frac{K_{CB}}{K_{CB} + K_{CF}} = \frac{2}{2 + 0.8} = 0.71$$

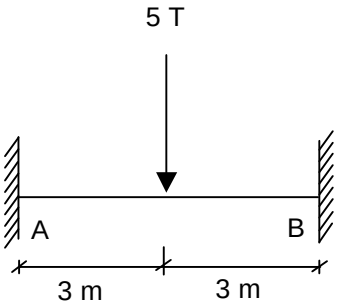
$$r_{CF} = \frac{K_{CF}}{K_{CB} + K_{CF}} = \frac{0.8}{2 + 0.8} = 0.29$$

2º. Calculamos los momentos y pares de empotramiento.



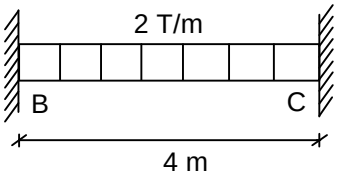
$$M_D = M_A = -\frac{q \cdot l^2}{12} = -\frac{1 \cdot 5^2}{12} = -2.08\ T \cdot m$$

$$m_D = +2.08\ T \cdot m$$
$$m_A = -2.08\ T \cdot m$$



$$M_A = M_B = -\frac{P \cdot l}{8} = -\frac{5 \cdot 6}{8} = -3.75\ T \cdot m$$

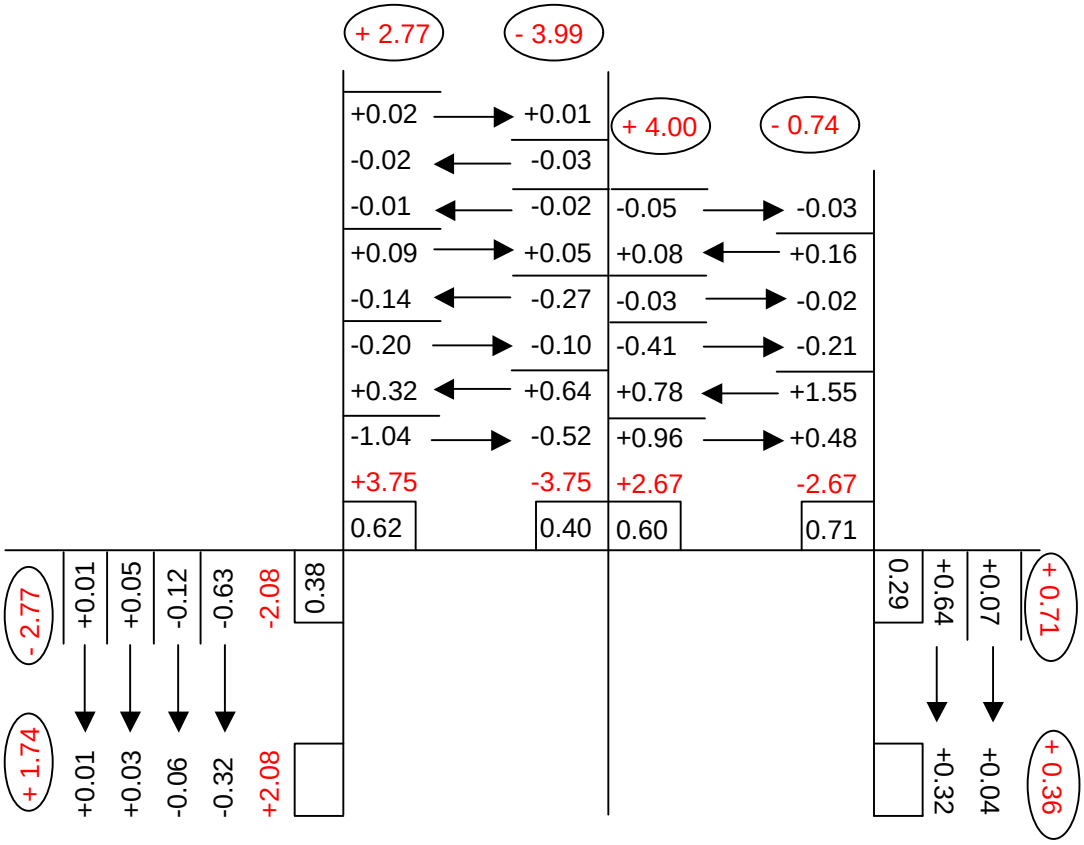
$$m_A = +3.75\ T \cdot m$$
$$m_B = -3.75\ T \cdot m$$



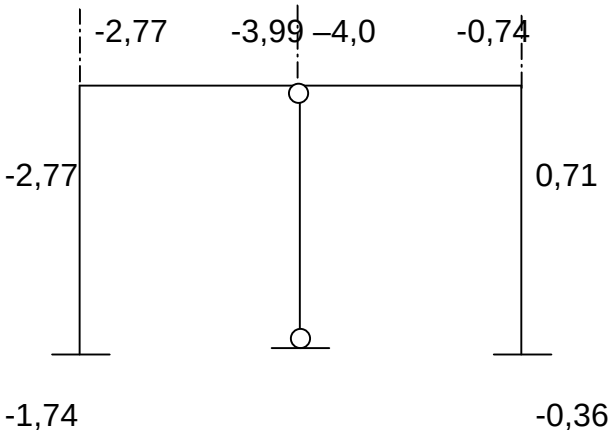
$$M_B = M_C = -\frac{q \cdot l^2}{12} = -\frac{2 \cdot 4^2}{12} = -2.67\ T \cdot m$$

$$m_B = +2.67\ T \cdot m$$
$$m_C = -2.67\ T \cdot m$$

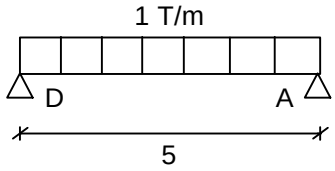
3º. Cross: Transmisiones.



- Momentos flectores en los nudos:

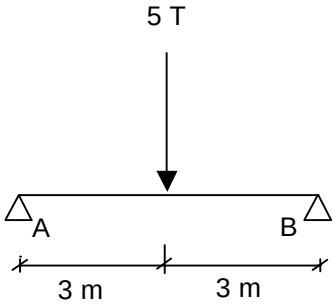


Momentos isostáticos



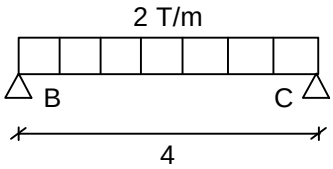
$$M_{máx\,i} = \frac{q \cdot l^2}{8} = \frac{1 \cdot 5^2}{8} = 3.13\ T \cdot m$$

$$M_{máx} = \frac{M_D + M_A}{2} + M_{máx_i} = \frac{-1.74 - 2.77}{2} + 3.13 = 0.87\ \ T \cdot m$$



$$M_{máx_i} = \frac{P \cdot l}{4} = \frac{5 \cdot 6}{4} = 7.5\ T \cdot m$$

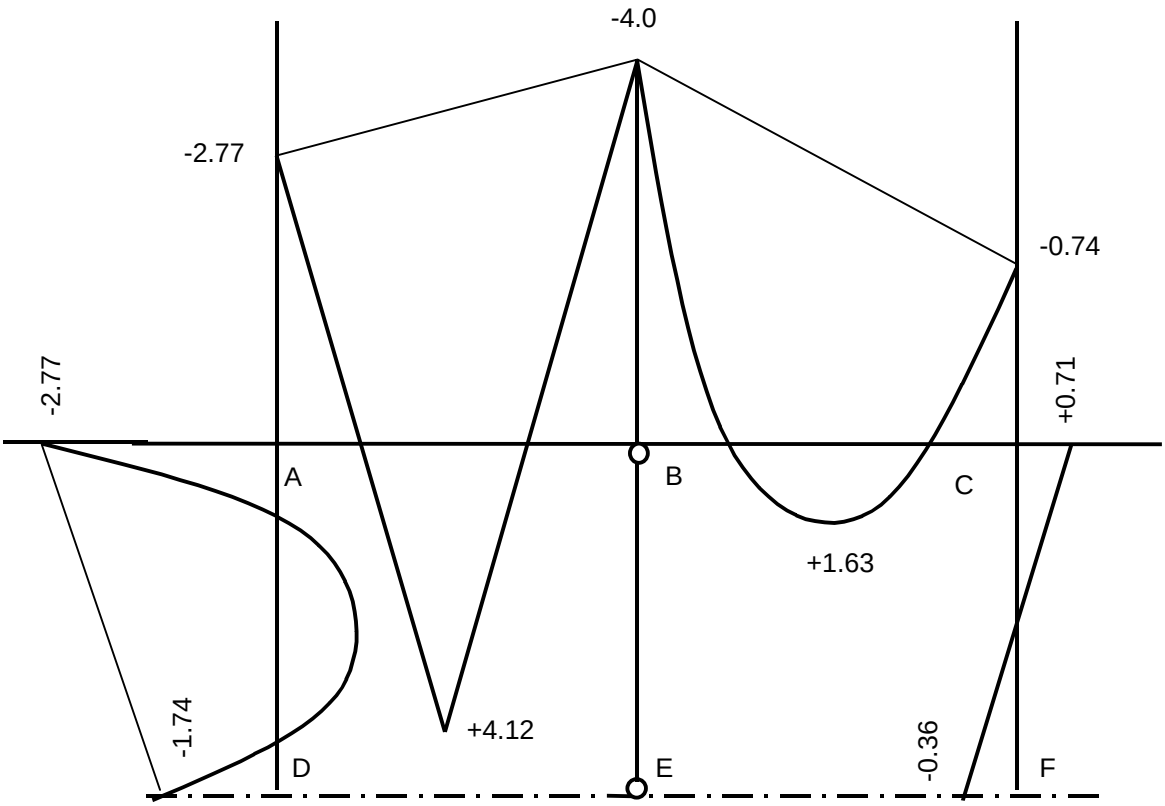
$$M_{máx} = \frac{-2.77 - 3.99}{2} + 7.5 = 4.12\ T \cdot m$$



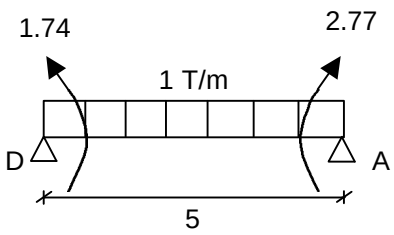
$$M_{máx_i} = \frac{q \cdot l^2}{8} = \frac{2 \cdot 4^2}{8} = 4\ T \cdot m$$

$$M_{máx} = \frac{M_B + M_C}{2} + M_{máx_i} = \frac{-4.0 - 0.74}{2} + 4 = 1.63\ T \cdot m$$

4º. Diagrama de momentos flectores.



5º. Cálculo de reacciones.



$$\sum M_A = 0$$

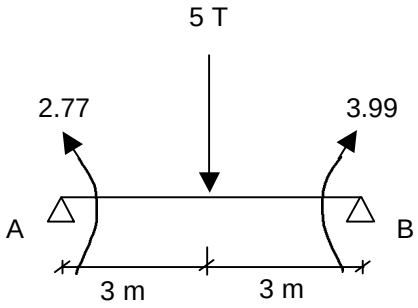
$$R_D \cdot 5 - 1.74 + 2.77 - 1 \cdot 5 \cdot \frac{5}{2} = 0$$

$$R_D = 2.29\ T$$

$$\sum M_D = 0$$

$$R_A \cdot 5 + 1.74 - 2.77 - 1 \cdot 5 \cdot \frac{5}{2} = 0$$

$$R_A = 2.71\,T$$



$$\sum M_B = 0$$

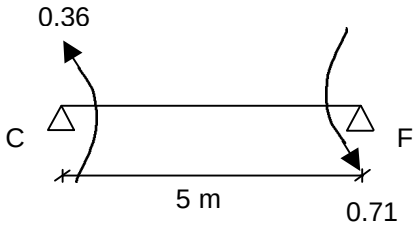
$$R_A \cdot 6 - 2.77 - 5 \cdot 3 + 3.99 = 0$$

$$R_A = 2.30\,T$$

$$\sum M_A = 0$$

$$R_B \cdot 6 - 3.99 + 2.77 - 5 \cdot 3 = 0$$

$$R_B = 2.70\,T$$



$$\sum M_C = 0$$

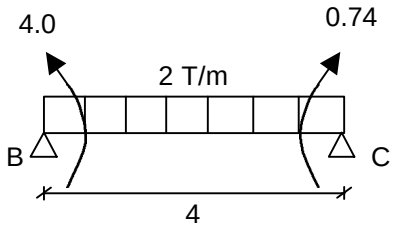
$$R_F \cdot 5 + 0.36 + 0.71 = 0$$

$$R_F = 0.21\,T$$

$$\sum M_F = 0$$

$$R_C \cdot 5 - 0.36 \quad 0.71 = 0$$

$$R_C = -0.21\,T$$



$$\sum M_B = 0$$

$$R_C \cdot 4 - 0.74 + 4.0 - 2 \cdot 4 \cdot \frac{4}{2} = 0$$

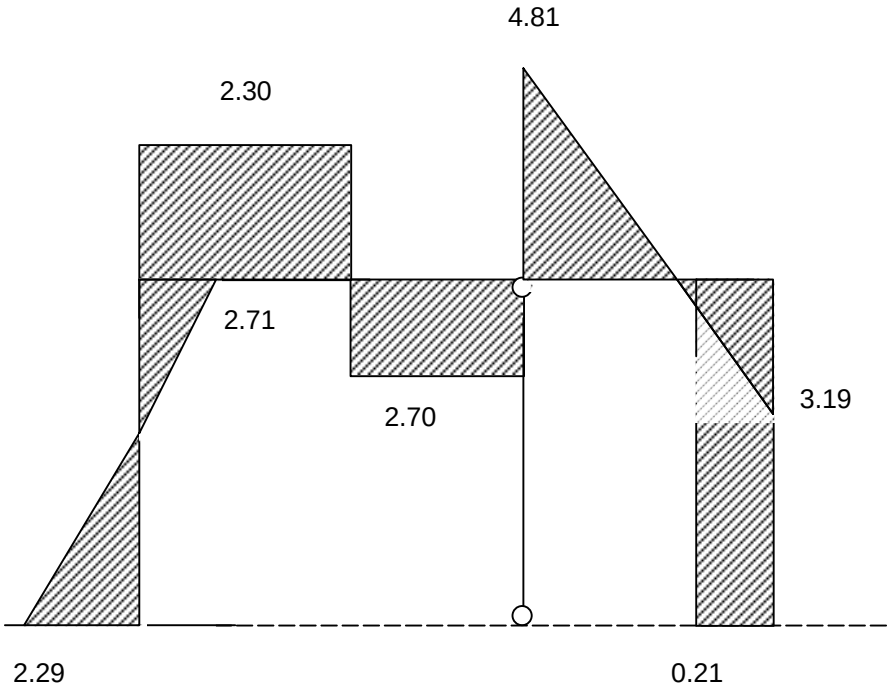
$$R_D = 3.19\,T$$

$$\sum M_C = 0$$

$$R_B \cdot 4 + 0.74 - 4.0 - 2 \cdot 4 \cdot \frac{4}{2} = 0$$

$$R_B = 4.81\,T$$

6º. Diagrama de esfuerzo cortante.



7º. Deformada.